647510 **BSCmaMJ205x-3C** **Seat No :________**

B.Sc. Semester - 3 (NEP-2020) Examination
OCT/NOV-2024
MATH: Linear Algebra -1 (Major-5)

Time: 2:00 Hours **Marks: 50**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two questions out of three. (10)

(A) if S is non empty subset of vector space V then prove that SPS is subspace of V.

(B) V is vector space. For $\overline{v_1}, \overline{v_2}, \overline{v_3}, \ldots \overline{v_n} \in V$ and SP$\{\overline{v_1}, \overline{v_2}, \overline{v_3}, \ldots \overline{v_n}\}$=V if $\overline{w_1}, \overline{w_2}, \overline{w_3}, \ldots \overline{w_n}$ are linearly independent vectors of V then prove that $m \leq n$.

(C) Check whether vector $(1,0,-1) \in$ SPA. where set A={(2,1,0),(-1,0,1),(0,1,2)}.

Que.2 Answer any two questions out of three. (10)

(A) State and prove Existence theorem for basis.

(B) State and prove Extension theorem for basis of vector space.

(C) $W_1=\{(x-y+z, x+y-z, y+z, 2x+4y+4z) \ / \ x,y,z \in R\}$
$W_2=\{(2y-4z+w, y+z, y-z, w) \ / \ y,z,w \in R\}$ be two subspaces of R^4. Find $\dim(W_1+W_2)$, $\dim W_1$, $\dim W_2$, $\dim(W_1 \cap W_2)$.

Que.3 Answer any two questions out of three. (10)

(A) Define rank and nullity of a linear transformation. Show that a linear transformation $T:U \to V$ is one-one iff $N_T=\{\theta\}$.

(B) $T:U \to V$ is any linear transformation then in usual Notation show that R_T is Subspace of V and N_T is subspace of U.

(C) State Rank-Nullity theorem and find N_T, R_T, n(T), r(T) for linear transformation $T:R^4 \to R^2$, T(a,b,c,d)=(a-b+c-d, 2a+b+3c+d) , Where $(a,b,c,d) \in R^4$.

Que.4 Answer any two questions out of three. (10)

(A) State and Prove Cauchy-schwartz's inequality for inner product space.

(B) Prove that any orthonormal set of vectors in an inner product space is Linearly independent.

(C) Let R^3 have the Euclidean inner product space. Use Gram-Schmidt process to transform the basis {(1,1,1),(-1,1,0),(1,2,1)} of R^3 in to orthonormal basis.

Que.5 Answer any two questions out of three. (10)

(A) **(i)** Examine W= {(a,b,c)/ a=3b} For subspace of R^3.
(ii) if set A= {(1,-2,5),(2,1,-1),(3,-1,b)} of vector space R^3 is Linearly independent Then find value of b.

(B) Extend set B= {(1,1,1),(2,0,0)} of vector space R^3 to form a base of R^3.

(C) **(i)** Check whether $T:R \to R^4$, where for $\forall x \in R$, T(x)= (x,2x,3x,4x) is a linear Transformation.
(ii) if u and v are vectors of real inner product space V such that ||u||=||v|| then Prove that (u+v).(u-v)=0.

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